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# Nonadiabatic Geometric Quantum Computation by Straightway Varying Parameters of Magnetic: A New Design

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**Abstract** The approach to implement nonadiabatic geometric quantum computation by controlling the magnetic fields is applied to construct single-qubit noncommutable geometric quantum gates. The results show that it is helpful for experimenters to realize the geometric quantum gates by adjusting the external parameters.

**Keywords** Nonadiabatic evolution · Geometric qubit · Quantum computation

## 1 Introduction

It is well known that the phase is a controllable factor in quantum system, which provides us conveniently to control the quantum physical system that is different from the classical physics. In micro-electron device, the phase effect of the current carriers is not considered. While in nanoelectronic device the amplitude and phase of the current carriers must be taken into account because the phase relations of the current carriers is remained after bouncy collisions. The present research results indicate that it is not enough to manipulate the quantum device only by controlling the number of electrons and it is mainly via controlling the phase of electron to realize a certain function. In addition, the geometric phase in quantum system does not rely on the dynamical properties of evolution process. It only relates to the topological properties of physical system, which will make it error tolerant to perform quantum information processing [1–4].

Recently, geometric phase has been attracting increasing interest because of its importance for understanding and implementing quantum computation in real physical systems

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[5–9]. Geometric quantum computation is a scheme intrinsically fault-tolerant and therefore resilient to certain types of computational errors. Theoretically, a pure geometric phase quantum gate can be achieved based on adiabatic geometric phase. But the adiabatic condition is not satisfied in many realistic cases. To solve this problem, A-A phase was suggested to complete geometric quantum gates. These geometric quantum gates have the faster gate-operation time and intrinsic geometric features. Up to now, there are two approaches to achieve the geometric gate: (1) driving the qubits to undergo appropriate adiabatic or nonadiabatic cyclic evolution and (2) displacing a harmonic oscillator along a closed path conditional on the state of the qubits [10–16]. However, the complexity of the controlling process will lead to more computation mistakes. Recently, Wang et al. [17] proposed a third scheme, in which it is pointed out that the parameters (amplitude and direction) for the magnetic field can be directly controlled by means of regulating the ratio of dynamic phase and geometric phase and consequently the geometric quantum computation can be realized. Obviously, this scheme adds advantage to implement quantum manipulation in laboratory.

In this work, we firstly propose strictly the dynamical evolution wave-function for a single qubit, and then discuss how to realize single quantum gate by manipulating the parameters for the magnetic field.

## 2 Wave Function of the System

Considering the Hamiltonian for a single-qubit system [17, 18]

$$H(t) = -\frac{1}{2}\Omega_1(\sigma_x \sin\theta \cos\omega t + \sigma_y \sin\theta \sin\omega t) - \frac{1}{2}\Omega_2\sigma_z \cos\theta, \tag{1}$$

where $\Omega_i = g\mu B_i/\hbar$ that are the gyromagnetic. $B_i$ $(i = 1, 2)$ and $\theta$ represent external controllable parameters and can be experimentally changed, and $\hat{\sigma}_i$ $(i = x, y, z)$ are Pauli spin operators.

Suppose that $|\Psi(t)\rangle$ is the state vector of the system that evolves according to the time-dependent Schrödinger equation ($\hbar = 1$)

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle. \tag{2}$$

By using the following canonical transform,

$$\Psi(t) = \exp\left(-i\frac{1}{2}\omega t\sigma_z\right)\tilde{\Psi}(t). \tag{3}$$

Equation (2) is rewritten as

$$i\hbar\frac{\partial}{\partial t}|\tilde{\Psi}(t)\rangle = \tilde{H}|\tilde{\Psi}(t)\rangle, \tag{4}$$

where the effective Hamiltonian is

$$\tilde{H} = -\frac{1}{2}\Omega(\sigma_x \sin\chi + \sigma_z \cos\chi), \tag{5}$$

with

$$\Omega = \sqrt{(\Omega_1 \sin\theta)^2 + (\Omega_2 \cos\theta + \omega)^2}, \tag{6}$$

$$\sin\chi = \frac{\Omega_1 \sin\theta}{\Omega}, \qquad \cos\chi = \frac{\Omega_2 \cos\theta + \omega}{\Omega}. \tag{7}$$

The eigenfunctions of the effective Hamiltonian $\tilde{H}$ are

$$\tilde{H}|\pm\rangle = -\frac{1}{2}k\Omega|\pm\rangle, \tag{8}$$

where $k = \pm 1$ and

$$|+\rangle = \cos\frac{\chi}{2}|0\rangle + \sin\frac{\chi}{2}|1\rangle, |-\rangle = -\sin\frac{\chi}{2}|0\rangle + \cos\frac{\chi}{2}|1\rangle.$$

From (3) and (5), the wave-function of the system can be expressed as

$$|\Psi(t)\rangle = U|\Psi(0)\rangle, \tag{9}$$

where

$$\begin{aligned} U(t) &= \exp(-i\omega t\sigma_z/2)\exp(-it\tilde{H}) \\ &= \begin{pmatrix} \left(\cos\frac{\Omega t}{2} - i\sin\frac{\Omega t}{2}\cos\chi\right)e^{-i\frac{\omega t}{2}} & -i\sin\frac{\Omega t}{2}\sin\chi e^{-i\frac{\omega t}{2}} \\ -i\sin\frac{\Omega t}{2}\sin\chi e^{i\frac{\omega t}{2}} & \left(\cos\frac{\Omega t}{2} + i\sin\frac{\Omega t}{2}\cos\chi\right)e^{i\frac{\omega t}{2}} \end{pmatrix}. \end{aligned}$$

## 3 Phases

Using these wave-functions, the corresponding geometric phases in time $t$ may be obtained as

$$\gamma_k(t) = \int_0^t d\tau\left\langle\Psi(\tau)|i\frac{\partial}{\partial\tau}|\Psi(\tau)\right\rangle = \frac{\omega t}{2}k(1-\cos\chi). \tag{10}$$

And the dynamical phase are given by

$$\beta_k(t) = -\int_0^t \langle\Psi(\tau)|H(\tau)|\Psi(\tau)\rangle d\tau = \frac{t}{2}k(\Omega_1\sin\theta\sin\chi + \Omega_2\cos\theta\cos\chi). \tag{11}$$

Thus the total phase is

$$\alpha_k(t) = \gamma_k(t) + \beta_k(t). \tag{12}$$

From (10)–(12), the total, geometric and dynamical phases of the system in the cyclic motion with period $T = 2\pi/\omega$ are respectively given by

$$\begin{aligned} \alpha_k(T) &= k\pi\left(1 - \frac{\Omega}{\omega}\right), \\ \gamma_k(T) &= k\pi(1-\cos\chi), \\ \beta_k(T) &= -k\pi\left(\frac{\Omega}{\omega} - \cos\chi\right). \end{aligned}$$

It is noted that the geometric phase exists at all times, not just when $H(t)$ has returned to its original form, which implies that the geometric phase is independent of the path and has a purely geometric property without the correction of the evolution. It is worth mentioning that the adiabatic geometric phase is only an approximation of the nonadiabatic geometrical phase. Though the geometric phase bears some resemblance to the Aharonov and Anandan phase, some differences exist, one of which lies in that here the cyclic condition is not needed. The geometric phase is of great significance since it is observable in the interference of two identically prepared systems.

## 4 A Design Scheme

From (10)–(12), Wang and co-workers (to see Ref. [17]) found that, when the relation

$$x = -\frac{(\Omega_1 \sin\theta)^2 + (\Omega_2 \cos\theta)^2 + \omega\Omega_2 \cos\theta}{\omega(\Omega - \omega - \Omega_2 \cos\theta)}, \tag{13}$$

is satisfied, the total, geometric and dynamical phases of the system in the cyclic motion with period $T = 2\pi/\omega$ satisfy

$$\beta_k = x\gamma_k, \qquad \alpha_k = (1+x)\gamma_k. \tag{14}$$

Thus, under the computational basis $\{|0\rangle, |1\rangle\}$, the unitary transformation $U(T)$, between the input and output state, can be written as

$$U(T) = \begin{pmatrix} e^{i(1+x)\gamma}\cos^2\frac{\chi}{2} + e^{-i(1+x)\gamma}\sin^2\frac{\chi}{2} & \frac{1}{2}\sin\chi\left(e^{i(1+x)\gamma} - e^{-i(1+x)\gamma}\right) \\ -\frac{1}{2}\sin\chi\left(e^{-i(1+x)\lambda} - e^{i(1+x)\gamma}\right) & e^{-i(1+x)\gamma}\cos^2\frac{\chi}{2} + e^{i(1+x)\gamma}\sin^2\frac{\chi}{2} \end{pmatrix}, \tag{15}$$

where

$$\gamma = \pi(1 - \cos\chi). \tag{16}$$

Qubit is the elementary component of quantum computer. Up to now, two kinds of phase gate have been proposed. One is based on geometric manipulation, namely drives the qubit to appropriate cyclic evolution that depends on the qubit state in order to obtain geometric phase. The other is based on the quantum computation of geometric manipulation, which is usually carried through in dark state. During the evolution, the quantum system is always in the eigenstate that the eigenvalue keeps zero (dark state), therefore there is no any dynamical phase shift. Wang and co-worker pointed out that the parameters (amplitude and direction) for the magnetic field can be directly controlled by regulating the ratio of dynamic phase and geometric phase and consequently the geometric quantum computation can be implemented. Unfortunately, they have not presented concrete experimental scheme.

As well-known, to achieve a set of universal quantum gates, we need to construct two noncommutable single-qubit gates and one nontrivial two-qubit gate. In this paper, we only discuss how to construct two noncommutable single-qubit gates [19, 20].

A. In the dark state

For $x = 0$, from (13), the parameters of an external controllable magnetic field satisfy:

$$(\Omega_1 \sin\theta)^2 + (\Omega_2 \cos\theta)^2 + \omega\Omega_2 \cos\theta = 0.$$

Furthermore, setting

$$\gamma = \pi(1 - \cos\chi) = \frac{3\pi}{2},$$

one finds

$$U(T)=\begin{pmatrix} -i\cos\chi & -i\sin\chi \\ -i\sin\chi & i\cos\chi \end{pmatrix}. \tag{17}$$

Making $x=0$ by controlling the magnetic field parameters, for example, with

$$\Omega_1=\Omega_2=-\omega\cos\theta,$$

we have

$$x=0.$$

Here, the quantum system is in dark state.

$$\cos\chi=\sin\theta=-\frac{1}{2}.$$

Thus, by choosing $\chi=2\pi/3$ and $\chi=4\pi/3$ in (17), respectively; it is straightforward to verify that $U(\chi=2\pi/3)$ and $U(\chi=4\pi/3)$ are noncommuting. Therefore, the two noncommutable single-qubit gates are constructed by directly varying the parameters of external controlled magnetic field.

B. $x\neq 0$

For this case, since the ratio of dynamic phase and geometric phase is non zero, we can construct two noncommutable single-qubit gates in a easier way in comparison with the dark state. For example, setting

$$(1+x)\gamma=-\frac{\pi}{2},$$

under the case of $\Omega_1=\Omega_2$, we choose the magnetic field parameters to satisfy

$$5\omega^2=4\Omega_1^2+8\omega\Omega_1\cos\theta,$$

and

$$\cos\chi=\frac{\Omega_1\cos\theta+\omega}{\sqrt{\omega^2+\Omega_1^2+2\omega\Omega_1\cos\theta}}. \tag{18}$$

Thus we choose the parameters $\chi=\pi/4$ and $\chi=\pi/3$ in (18) to construct two noncommuting geometric quantum gates. it is easy to verify that $U(\chi=\pi/4)$ and $U(\chi=\pi/3)$ are noncommuting. The two noncommutable single-qubit gates can be controlled by straightway varying parameters of the external controlled magnetic field.

## 5 Conclusions

In conclusion, based on the dynamically evolving wave-function of a single quantum qubit, we know how to control the magnetic field parameters (amplitude and direction) and implement geometric quantum computation by regulating the ratio of dynamic phase and geometric phase. It is known that the approach is remarkable in that it achieves completely by manipulating the magnetic field parameters and requires no rotation operation unlike the conventional geometric gates or the unconventional geometric gates.

Obviously, at $x = 0$, the system is in dark state under the cyclic evolution. The result is similar to conventional geometric quantum computation, where the dynamical phase was cancelled by rotating operations in single-loop and multi-loop approaches. At the case of $x \neq 0$, the dynamical phase shift occurs. This result is similar to unconventionally geometric quantum computation, where ones used the global geometric features in the rotating frame at the cavity frequency and did not distinguish both the total phase and geometric phase. Under our approach, an appropriate path is not necessary for Hamiltonian. It is realized just by regulating the magnetic field parameters quantum manipulation, which is more feasible in experiment and therefore attractive for experimental measurement.

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## References

1. Zanardi, P., Rasetti, M.: Holonomic quantum computation. Phys. Lett. A **264**, 94 (1999)
2. Jones, J.A., Vedral, V., Ekert, A., Castagnoli, G.: Geometric quantum computation using nuclear magnetic resonance. Nature (London) **403**, 869 (2000)
3. Duan, L.M., Cirac, J.I., Zoller, P.: Geometric manipulation of trapped ions for quantum computation. Science **292**, 1695 (2001)
4. Falci, G., Fazio, R., Palma, G.M., Siewert, J., Vedral, V.: Detection of geometrical phases in superconducting nanocircuits. Nature (London) **407**, 355 (2000)
5. Wang, Z.S., Kwek, L.C., Lai, C.H., Oh, C.H.: Geometric phase and entanglement from massive spin-1 particles. Eur. Phys. J. D **33**, 285 (2005)
6. Wang, Z.S., Kwek, L.C., Lai, C.H., Oh, C.H.: Geometric phase in open two-level system. Europhys. Lett. **74**, 958 (2006)
7. Wang, Z.S., Wu, Ch., Feng, X.-L., Kwek, L.C., Lai, C.H., Oh, C.H.: Effects of a squeezed-vacuum reservoir on geometric phase. Phys. Rev. A **75**, 024102 (2007)
8. Wang, Z.S., Kwek, L.C., Lai, C.H., Oh, C.H.: Quantum tunneling via quantum geometric phase. Phys. Lett. A **359**, 608 (2006)
9. Wang, Z.S., Wu, Ch., Feng, X.-L., Kwek, L.C., Lai, C.H., Oh, C.H., Vedral, V.: Geometric phase induced by quantum nonlocality. Phys. Lett. A **372**, 775 (2008)
10. Pachos, J., Zanardi, P., Rasetti, M.: Non-Abelian Berry connections for quantum computation. Phys. Rev. A **61**, 010305 (2000)
11. Wang, Z.S.: Geometric quantum computation and dynamical invariant operators. Phys. Rev. A **79**, 024304 (2009)
12. Wang, Z.S., Liou, G.Q., Ji, Y.H.: Noncyclic geometric quantum computation in a nuclear-magnetic-resonance system. Phys. Rev. A **79**, 054301 (2009)
13. Leibfried, D., DeMarco, B., Meyer, V., Lucas, D., Barrett, M., Britton, J., Itano, W.M., Jelenkovic, B., Langer, C., Rosenband, T., Wineland, D.J.: Experimental demonstration of a robust, high-fidelity geometric two ion-qubit phase gate. Nature **422**, 412 (2003)
14. Zhu, S.L., Wang, Z.D.: Unconventional geometric quantum computation. Phys. Rev. Lett. **91**, 187902 (2003)
15. Zheng, S.B.: Unconventional geometric quantum phase gates with a cavity QED system. Phys. Rev. A **70**, 052320 (2004)
16. Chen, C.Y., Feng, M., Zhang, X.L., Gao, K.L.: Strong-driving-assisted unconventional geometric logic gate in cavity QED. Phys. Rev. A **73**, 032344 (2006)
17. Wang, Z.S., Wu, Ch., Feng, X.-L., Kwek, L.C., Lai, C.H., Oh, C.H., Vedrel, V.: Nonadiabatic geometric quantum computation. Phys. Rev. A **76**, 044303 (2007)
18. Zhu, S.L., Wang, Z.D.: Implementation of universal quantum gates based on nonadiabatic geometric phases. Phys. Rev. Lett. **89**, 097902 (2002)
19. Zhang, X.D., Zhu, S.L., Hu, L., Wang, Z.D.: Nonadiabatic geometric quantum computation using a single-loop scenario. Phys. Rev. A **71**, 014302 (2005)
20. Du, J.F., Zou, P., Wang, Z.D.: Experimental implementation of high-fidelity unconventional geometric quantum gates using an NMR interferometer. Phys. Rev. A **74**, 020302 (2006)